

Quantum non-demolition measurement of Werner state

Jia-sen Jin, Chang-shui Yu,* Pei Pei, and He-shan Song†

*School of Physics and Optoelectronic Technology,
Dalian University of Technology, Dalian 116024 China*

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We propose a theoretical scheme of quantum non-demolition measurement of two-qubit Werner state. We discuss our scheme with the two qubits restricted in a local place and then extend the scheme to the case in which two qubits are separated. We also consider the experimental realization of our scheme based on cavity quantum electrodynamics. It is very interesting that our scheme is robust against the dissipative effects introduced by the probe process. We also give a brief interpretation of our scheme finally.

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I. INTRODUCTION

Performing a measurement on a quantum system will introduce a disturbance due to the unavoidable back action of the measurement on the detected observable, so that the successive measurements of the same observable yield different outcomes. However, the quantum nondemolition (QND) measurements [1-4] allow us to reproduce identical outcome when the measurements of the same observable are repeated. A measurement of an observable is called QND measurement if the system is not polluted during the time evolution and the observable is back-action evading during the measurement process, which means that the observable being measured commutes with the interaction Hamiltonian. Once the requirements of QND measurement are satisfied, we can measure an observable of a certain quantum system repeatedly and obtain predictable results.

Since the idea of QND measurement was introduced [5,6], it has been widely investigated in the last two decades. Generally, a QND measurement of an observable A_s in a quantum system is performed by detecting a change in an observable A_p of the probe system which is coupled to the system being measured. Based on this idea, numerous schemes for QND measurements have been proposed theoretically and experimentally in the fields of quantum optics [7-10], atomic physics [11-13], and cavity quantum electrodynamics (QED) systems [14-17]. For example, in the quantum optics domain, the photon number of a signal beam can be nondestructively measured by detecting the phase shift on a probe beam which is coupled to the signal beam in a nonlinearity medium [10]. However, most schemes of QND measurement only focused on a certain observable of a quantum system, but the whole state of the system cannot be ascertained, so we can say those schemes are the QND measurement of the observable rather than the quantum state.

In this paper, we present a QND measurement for the two-qubit Werner state, by which we can completely obtain all the information of the state, since the Werner state [18] is a family of a one-parameter state which is the mixture of the

maximally mixed state and the pure maximally entangled state with the mixing proportion parameterized by a real parameter $x \in [0, 1]$. In our scheme, the measurement is performed not directly on the Werner state but on a probe qubit. In the process of QND measurement, the Werner state is not disturbed; it always remains in the initial state. The distinguished advantages of our scheme are : (1) The scheme can be used to acquire the whole information of a quantum state rather than only an observable of a quantum system, in particular, it is also suitable for the state with two separated qubits; (2) In the whole processing of measurements, all the properties of the Werner state is preserved, since the state is not disturbed; (3) our scheme is robust against the dissipative effects introduced by the probe process. This paper is organized as follows. In sec. II, we will illustrate our scheme with the two qubits in local and separated places, respectively. In Sec. III, we discuss our scheme in an experimental scenario and present the principles of our scheme for QND measurement. The conclusion is drawn finally.

II. SCHEMES FOR QND MEASUREMENT OF WERNER STATE

In this section we will illustrate our scheme explicitly. Suppose that a two-qubit system is prepared in the following Werner state, initially,

$$\rho_{12} = \frac{1-x}{4} \mathbb{I}_{12} + x |\Psi^-\rangle_{12} \langle \Psi^-|, \quad (1)$$

where $|\Psi^-\rangle_{12} = (|1\rangle_1 |0\rangle_2 - |0\rangle_1 |1\rangle_2) / \sqrt{2}$ is the well-known Bell state, \mathbb{I}_{12} is the identity matrix of two-qubit Hilbert space, and $|0\rangle$ and $|1\rangle$ are the computational basis with the forms

$$|0\rangle_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, |1\rangle_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (2)$$

The subscript denotes the label of the qubit. In order to ascertain the value of x , we need a probe qubit (labeled 3) which is prepared in the state $|0\rangle_3$. Thus the state of the joint system consisting of three qubits is given $\rho = \rho_{12} \otimes |0\rangle_3 \langle 0|$. We perform an unitary operation on the joint system; the unitary

*Electronic address: quaninformation@sina.com; ycs@dlut.edu.cn

†Electronic address: hssong@dlut.edu.cn

operator has the following form:

$$U = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & -i & 0 & -i & -1 & 0 \\ 0 & 1 & -i & 0 & -i & 0 & 0 & -1 \\ 0 & -i & 1 & 0 & -1 & 0 & 0 & -i \\ -i & 0 & 0 & 1 & 0 & -1 & -i & 0 \\ 0 & -i & -1 & 0 & 1 & 0 & 0 & -i \\ -i & 0 & 0 & -1 & 0 & 1 & -i & 0 \\ -1 & 0 & 0 & -i & 0 & -i & 1 & 0 \\ 0 & -1 & -i & 0 & -i & 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

As a result, the state of the joint system evolves to $\rho' = U\rho U^\dagger$ (we use *prime* to denote that the state undergoes unitary operation hereinafter). Tracing over qubit 3, we can obtain the reduced density matrix of qubits 1 and 2 as follows:

$$\rho'_{12} = \frac{1}{4} \begin{pmatrix} 1-x & 0 & 0 & 0 \\ 0 & 1+x & -2x & 0 \\ 0 & -2x & 1+x & 0 \\ 0 & 0 & 0 & 1-x \end{pmatrix}. \quad (4)$$

Note that the state of qubits 1 and 2 is the same to the initial state, that is to say the Werner state given in Eq. (1) is unchanged under the unitary operation U . We can also obtain the reduced density matrix of qubit 3 by tracing over qubits 1 and 2 as follows

$$\rho'_3 = \frac{1}{2} \begin{pmatrix} 1-x & 0 \\ 0 & 1+x \end{pmatrix}. \quad (5)$$

It is interesting that the expression of ρ'_3 includes the parameter x , which means the information of the two-qubit system being measured is transferred to qubit 3. Consequently, we can obtain the value of x by performing projective measurements on qubit 3 as well as keeping the state of qubits 1 and 2 undisturbed; the QND measurement of a two-qubit Werner state is accomplished.

In fact, our scheme can also be used for the other types of Werner states, [*i.e.*, the Werner state is a mixture of the maximally mixed state with one of the other three Bell states: $|\Phi^-\rangle_{12} = (|1\rangle_1|1\rangle_2 - |0\rangle_1|0\rangle_2)/\sqrt{2}$, $|\Psi^+\rangle_{12} = (|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2)/\sqrt{2}$, or $|\Phi^+\rangle_{12} = (|1\rangle_1|1\rangle_2 + |0\rangle_1|0\rangle_2)/\sqrt{2}$]. For the case of $|\Phi^-\rangle_{12}$, we find that the unitary operator U given in Eq. (3) is still feasible for the QND measurement of Werner state. For the cases of $|\Psi^+\rangle_{12}$ and $|\Phi^+\rangle_{12}$ the unitary operator U is not suitable any more, however, we can send qubits 1 and 2 passing through a controlled phase-flip gate to make the transformations $|\Psi^+\rangle_{12} \rightarrow |\Psi^-\rangle_{12}$ and $|\Phi^+\rangle_{12} \rightarrow |\Phi^-\rangle_{12}$, in this way we can obtain the value of x with the QND measurement mentioned previously, and then perform an inverse transformation to transfer $|\Psi^-\rangle_{12}$ or $|\Phi^-\rangle_{12}$ back to the original form.

The previously mentioned scheme implies that the Werner state is in the same place, or more precisely speaking, it needs the interaction of three qubits. Next we would like to emphasize that our scheme can also be suitable for the Werner state separately shared. Let us look back to the unitary operator U given in Eq. (3). This operator is essentially a tripartite manipulation on qubits, and it can be formally factorized as

$U = (U_{13} \otimes \mathbb{I}_2)(\mathbb{I}_1 \otimes U_{23})$, with U_{13} and U_{23} having the following form

$$U_{13} = U_{23} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

Based on this factorization we can accomplish the QND measurement of the Werner state by performing bipartite manipulation U_{13} on qubits 1 and 3, and U_{23} on qubits 2 and 3, sequentially. That is to say, even if qubits 1 and 2 are separated into two distant places, we can still implement the QND measurement for the Werner state, the procedures are given as follows. Suppose that qubit 1 together with qubit 3 locates at place A and qubit 2 locates at place B, the two-qubit Werner state is given by Eq. (1) and qubit 3 is in state $|0\rangle_3$, initially. First, we perform unitary operation U_{13} at place A, next send qubit 3 to place B, and then we perform the operation U_{23} on qubits 2 and 3. Finally, we perform the projective measurements on qubit 3 to ascertain the value of x and accomplish the QND measurement of the Werner state.

III. QND MEASUREMENT OF WERNER STATE IN EXPERIMENTAL SCENARIO

In the following we will discuss our scheme based on a cavity QED system in order to show the experimental realization of the QND measurement of the Werner state. We consider that two identical two-level atoms 1 and 2 (with excited state $|e\rangle$ and ground state $|g\rangle$) are trapped in two optical cavities A and B, respectively. These two cavities are arranged to be crossed as shown in Fig. 1 (a). The probe atom 3 is trapped in the overlapped region of the two cavity fields, and is additionally driven by an external classical field with coupling constant Ω . All the atoms couple to their corresponding cavity modes with the same coupling constant g . The Hamiltonian governing the joint system is given by (in the following $\hbar = 1$)

$$\begin{aligned} H = & \sum_{i=1,2,3} \omega_i \sigma_i^+ \sigma_i^- + \nu(a^\dagger a + b^\dagger b) \\ & + g(\sigma_1^+ a + \sigma_2^+ b + \sigma_3^+ a + \sigma_3^+ b + \text{H.c.}) \\ & + \Omega(e^{-i\omega_L t} \sigma_3^+ + e^{i\omega_L t} \sigma_3^-), \end{aligned} \quad (7)$$

where ω_i , ν , and ω_L are the frequencies of the atomic transition, the cavity modes, and classical field, respectively; $\sigma_i^+ = |e\rangle_i \langle g|$ and $\sigma_i^- = |g\rangle_i \langle e|$ are the raising and lowering operators of the i th atom; a and b are the annihilation operators of the cavity A and B, respectively. In addition, we set $\delta = \omega_3 - \nu$. If there are no photons in both cavities, under the large detuning condition $\delta \gg g$, we can adiabatically eliminate the cavity modes and obtain the following Hamiltonian in a proper rotating frame,

$$H' = \lambda(\sigma_3^+ \sigma_1^- + \sigma_3^+ \sigma_2^- + \text{H.c.}) + \Omega(\sigma_3^+ + \sigma_3^-). \quad (8)$$

The effective coupling constant between atoms is given as follows:

$$\lambda = \omega_3 - \omega_{1(2)} = g^2/\delta. \quad (9)$$

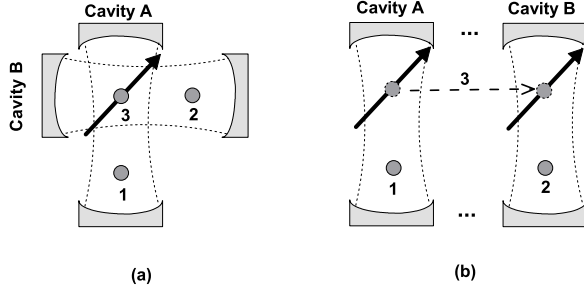


FIG. 1: (a) Schematic illustration for QND measurement. Two identical two-level atoms, previously prepared in the Werner state, are trapped in two crossed optical cavities, respectively. The probe atom is located at the overlapped region of cavity fields and additionally driven by a strong classical field. (b) Schematic illustration for QND measurement when the cavities are separated. The probe atom first interacts with atom 1 in cavity A and is then sent to cavity B to interact with atom 2. While the atom is interacting with atom 1 or 2 in each cavity, it is also driven by a strong classical field.

Further, in the strong driving regime $\Omega \gg \lambda$ [19,20], we can obtain the effective Hamiltonian of the joint system as follows,

$$H_{\text{eff}} = \frac{\lambda}{2}(\sigma_1^x + \sigma_2^x)\sigma_3^x, \quad (10)$$

where $\sigma_i^x = \sigma_i^+ + \sigma_i^-$ is the Pauli matrix of the i th atom.

Initially, the joint state of the atoms 1 and 2 is prepared in the Werner state given in Eq. (1) (the atomic Werner state can be generated in the cavity QED system [21]), and the probe atom 3 is prepared in the ground state. It is obvious to see that the unitary time-evolution operator $e^{-iH_{\text{eff}}t}$ is in accordance with the unitary operator given in Eq. (3) at time points $t = \frac{(2n+1)\pi}{2\lambda}$ ($n = 0, 1, 2, \dots$). In order to gain further insight into this model, we shall calculate the time evolution of the joint system. It is shown from the analytical results that the joint state of atoms 1 and 2 is time-independent and the probing atomic state varies periodically with the evolution time. That is to say, the two-qubit Werner state will remain unchanged during the time evolution and the probing atomic state has the following form:

$$\rho_3' = \frac{1}{2} \begin{pmatrix} (1-x)\sin^2 \lambda t & 0 \\ 0 & 1 + \cos^2 \lambda t + x \sin^2 \lambda t \end{pmatrix}. \quad (11)$$

As shown in Eq. (11), the state of the probe atom carries the information of the Werner state during the time evolution, except for some specific time points $t = \frac{n\pi}{2\lambda}$ ($n = 0, 1, 2, \dots$) at which the probing atomic state falls to $|g\rangle_3$ and the total system evolves back to the initial state. Therefore, we can perform the projective measurements on qubit 3 at the arbitrary time that the state of the total system differs from the initial state to acquire the information of the Werner state. Moreover, we emphasize that the Werner state is unchanged during the time evolution, we need only one copy of the Werner state in the whole processing of the QND measurement.

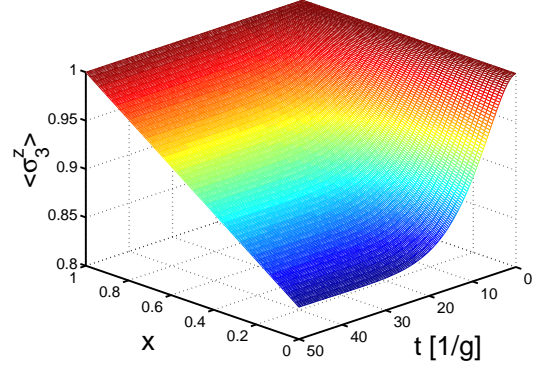


FIG. 2: (Color online) Time evolution of the population of atomic ground state as a function of parameter x and time t . The spontaneous emission rate is chosen as $\gamma = 0.1g$.

Next we will investigate the influence of dissipative process on the QND measurement. We only focus on the effects of the atomic spontaneous emission of probe atom and ignore those effects of the atoms being measured, since if we take into account the atomic spontaneous emissions of atoms 1 and 2, the Werner state will be destroyed naturally no matter whether we have measured it or not. Considering the atomic spontaneous emission of the probe atom, the time evolution of the joint system is described by the following master equation

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + \gamma(2\sigma_3^- \rho \sigma_3^+ - \sigma_3^+ \sigma_3^- \rho - \rho \sigma_3^+ \sigma_3^-), \quad (12)$$

where γ is the atomic spontaneous emission rate and H_{eff} is defined as Eq. (10). Initially, atoms 1 and 2 are prepared in the Werner state given in Eq. (1) and atom 3 is prepared in the ground state. We have numerically solved the master equation and found some interesting results. On the one hand, the Werner state still remains unchanged even if the atomic spontaneous emission is taken into account; on the other hand, the probing atomic state evolves to a steady state which, in particular, depends on the value of x . We have plotted the quantity of $\langle \sigma_3^z \rangle$ as a function of the parameter x and time t in Fig. 2. From the figure, we can find that the steady state value of $\langle \sigma_3^z \rangle$ shows a perfect linearity about the parameter x which provides us a novel idea to implement the QND measurement of the Werner state, the procedures are as follows. Introduce the atomic spontaneous emission of the probe atoms, and drive the probe atom with a strong classical field for a sufficiently long time, then perform a measurement of $\langle \sigma_3^z \rangle$, according to the linear relationship between $\langle \sigma_3^z \rangle$ and x shown in Fig. 2, we can acquire the information of the Werner state without destroying it.

The Hamiltonian given in Eq. (10) has two parts which commutes with each other, so the unitary time evolution operator can be decomposed into two independent unitary operators acting on atoms 1 and 3, and on atoms 2 and 3, respectively, as we discussed in Sec II. Thus, if atoms 1 and 2 are trapped in two separated cavities, even if very distant, it is still possible to perform the QND measurement for Werner state; the schematic is shown in Fig. 1(b). Suppose that atom

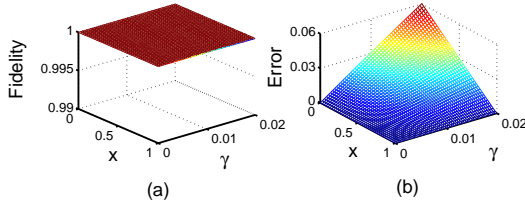


FIG. 3: (Color online) (a) Fidelity of the Werner state as a function of parameter x and spontaneous emission rate γ (in units of g). (b) Error of x , defined as $\delta x = |x - x'|$, as a function of parameter x and spontaneous emission γ (in units of g). The interaction times in each cavity are chosen as $t_1 = t_2 = \pi/2g$.

1 and the probe atom 3 are trapped in cavity A, firstly. A strong classical field is driving on the probe atom. Based on the approximation technique mentioned previously, the effective Hamiltonian describing the interactions between atoms 1 and 3 is given as follows

$$H_1 = \frac{\lambda_1}{2} \sigma_1^x \sigma_3^x, \quad (13)$$

where λ_1 is the effective coupling constant between atoms 1 and 3. The unitary time-evolution operator $e^{-iH_1 t}$ coincides with the unitary operator given in Eq. (6) at time $t = \frac{(2n+1)\pi}{2\lambda_1} (n = 0, 1, 2, \dots)$. And then we send the probe atom 3 from cavity A to cavity B, the interaction mechanism in cavity B is similar to that in cavity A; the effective Hamiltonian is given by

$$H_2 = \frac{\lambda_2}{2} \sigma_2^x \sigma_3^x, \quad (14)$$

where λ_2 is the effective coupling constant between atoms 2 and 3. The unitary time-evolution operator $e^{-iH_2 t}$ coincides with the unitary operator given in Eq. (5) at time $t = \frac{(2n+1)\pi}{2\lambda_2} (n = 0, 1, 2, \dots)$. Therefore we can realize the two local operations by controlling the interaction times in each cavity. After the two local operations, we perform the projective measurements on the probe atom to obtain the value of x and in turn we can acquire the whole information of the Werner state.

We have also examined the time evolution of the joint system, we assume the interaction times in cavity A and in cavity B to be t_1 , and t_2 , respectively. We find that if interaction time t_2 satisfies $t_2 = \frac{\lambda_1}{\lambda_2} t_1 + \frac{2n\pi}{\lambda_2} (n = 0, 1, 2, \dots)$, the QND measurement of the Werner state will succeed, namely it is not necessary to require the coupling constants in different cavities to be identical which relaxes the conditions for the experimental realization. Taking into account the atomic spontaneous emission, we have plotted the fidelity of the Werner state and the error δx after the measurement as functions of parameter x and the spontaneous emission rate γ in Figs. 3 (a) and 3(b), respectively. The error δx is defined as the absolute value of the discrepancy between the measured value x' and the true value of x , *i.e.* $\delta x = |x - x'|$. The interaction times in each cavity are chosen as $t_1 = t_2 = \pi/2g$. From Fig. 3(a), one can find that the atomic spontaneous emission has almost no influence on the fidelity of the Werner state. From Fig. 3(b),

we find that the absolute error δx is dependent on the values of x and γ . For a small x (close to 0) and a large γ , δx is relatively larger, whereas for a large x (close to 1) and a small γ , δx is negligible. On the whole, the error is less than 0.06, in this sense, we say our scheme is robust against the atomic spontaneous emission.

Now let us briefly discuss the principles of the QND measurement based on this model. Note that the Hamiltonian given in Eq. (10) has four dark states $\phi_1 = |\Psi^-\rangle_{12} \otimes |g\rangle_3$, $\phi_2 = |\Phi^-\rangle_{12} \otimes |g\rangle_3$, $\phi_3 = |\Psi^-\rangle_{12} \otimes |e\rangle_3$, and $\phi_4 = |\Phi^-\rangle_{12} \otimes |e\rangle_3$. The Werner state is mixed by two components: the maximally mixed state and the Bell state. The maximally mixed state component in the Werner state is unchanged under the joint unitary operation, the reason can be interpreted as follows. The maximally mixed state can be written as a mixture of the four Bell states. The terms $|\Psi_{12}^-\rangle$ and $|\Phi_{12}^-\rangle$ together with qubit 3 compose the dark states of the system, which are naturally unchanged during the time evolution; on the other hand, the effects of the time evolutions of $|\Psi_{12}^+\rangle$ and $|\Phi_{12}^+\rangle$ together with qubit 3 counteract each other, which are unchanged in the time evolution, too. Therefore, the total effect is that the maximally mixed state is unchanged under the unitary operation U , so if the Bell state component in the Werner state is $|\Psi_{12}^-\rangle$ or $|\Phi_{12}^-\rangle$, the Werner state will remain unchanged, this makes our scheme to be state nondestructive. Furthermore, the time evolution of the state of qubit 3 is completely frozen by the Bell state component and completely unrestricted by the maximally mixed state, so the population of qubit 3 reveals the value of x effectively.

IV. CONCLUSION AND DISCUSSION

In conclusion, we have presented a scheme for QND measurement of the two-qubit Werner state. Our scheme can be used to acquire the whole information of the Werner state, and in the whole processing the state is undisturbed. Moreover, we discussed our scheme in the frame of the cavity QED system. It is very interesting that our scheme is robust against the influence of atomic spontaneous emission. In particular, if the qubits of the Werner state can interact with the probe atom simultaneously, the influence of the atomic spontaneous emission can be completely eliminated. In addition, we show that if the Bell state component of the Werner state is $|\Psi_{12}^-\rangle$ or $|\Phi_{12}^-\rangle$, we can perform the QND measurement directly; if the Bell state component of the Werner state is $|\Psi_{12}^+\rangle$ or $|\Phi_{12}^+\rangle$, we should first transform them to the state $|\Psi_{12}^-\rangle$ or $|\Phi_{12}^-\rangle$, and then perform the QND measurement. Finally, we would like to emphasize that our scheme can also be realized in other physical systems, such as in the spin chain system [22] or in Josephson junction [23], and so on.

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